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10MAT31

Third Semester B.E. Degree Examination, Aug./Sept. 2020
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Find the Fourier series for the function f(x) = x(2π - x) over the interval (0, 2π) and hence deduce that $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$. **(07 Marks)**

b. Obtain the half range Fourier cosine series for the function :

$$f(x) = \begin{cases} Kx \ ; & 0 \le x \le \ell/2 \\ K(\ell - x); & \ell/2 \le x \le \ell \end{cases}$$

Where K is a constant. **(07 Marks)**

c. Obtain the constant term and co-efficient of first cosine and sine terms in the expansion of y from the following table :

x	0	π/3	2π/3	π	4π/3	5π/3	2π
y	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

(06 Marks)

2 a. Find the Fourier transform of the function :

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$ and hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$. **(07 Marks)**

b. Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}, a > 0, x > 0$. **(07 Marks)**

c. Find the Fourier cosine transform of $f(x) = e^{-ax}$; a > 0. **(06 Marks)**

3 a. Obtain the various possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. **(07 Marks)**

b. Obtain the D'Alembert's solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions, u(x, 0) = f(x) and $\frac{\partial u}{\partial t}(x,0) = 0$. **(07 Marks)**

c. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < \pi$, under the conditions :

i) u(0, t) = u(π, t) = 0

ii) u(x, 0) = 0

iii) $\frac{\partial u}{\partial t}(x,0) = A(\sin x - \sin 2x)$, A ≠ 0. **(06 Marks)**

4 a. Fit a parabola $y = ax^2 + bx + c$ by the method of least squares for the following data :

x	0	1	2	3	4
y	1	0.8	1.3	2.5	6.3

(07 Marks)

b. Minimize $Z = 5x + 4y$
Subject to the constraints : $x + 2y \geq 10$
$x + y \geq 8$
$2x + y \geq 12$
$x \geq 0, y \geq 0$
by Graphical method. (06 Marks)

c. Use Simplex method to
Maximize $Z = 2x + 4y$
Subject to the constraints $3x + y \leq 22$
$2x + 3y \leq 24$
$x \geq 0, y \geq 0.$ (07 Marks)

PART – B

5 a. Using the Newton's–Raphson method, find an approximate root of the equation $x\log_{10}x = 1.2$ that lies near 2.5 correct to four decimal places. (07 Marks)

b. Apply the Gauss – Seidel iterative method to solve the system of equations :
$5x + 2y + z = 12$
$x + 4y + 2z = 15$
$x + 2y + 5z = 20$
carryout four iternations, taking the initial approximation to the solution as (1, 0, 3). (07 Marks)

c. Using Rayleigh's power method to find the largest eigen value and the corresponding eigen vector of the matrix :

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$ taking $[1, 1, 1]^T$ as the initial eigen vector. Perform five iterations.

(06 Marks)

6 a. A function $y = f(x)$ is given by the following table :

x	1.0	1.2	1.4	1.6	1.8	2.0
y = f(x)	0.00	0.128	0.544	1.296	2.432	4.00

Find f(1.1) using suitable interpolation formula. (07 Marks)

b. Fit a polynomial for the following data, using Newton's divided difference formula. Hence find f(8) and f(15).

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

(07 Marks)

c. By using Simpson's $(3/8)^{th}$ rule with $h = 0.2$, find the approximate area under the curve $y = \frac{x^2 - 1}{x^2 + 1}$ between the orinates $x = 1$ and $x = 2.8$. (06 Marks)

7 a. Solve Laplace's equation $u_{xx} + u_{yy} = 0$ for the following square Mesh with boundary values as shown in the following Fig.Q7(a).

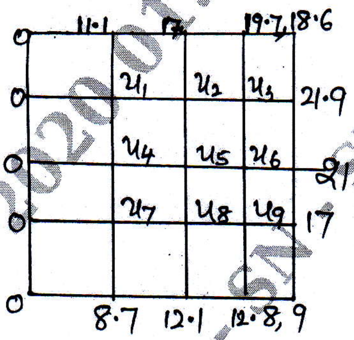

Fig.Q7(a) **(07 Marks)**

b. Solve the wave equation $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ given $u(0, t) = u(5, t) = 0$, $t \geq 0$, $u(x, 0) = x(5 - x)$, $\frac{\partial u}{\partial t}(x,0) = 0$, $0 < x < 5$. Find u at $t = 2$ given $h = 1$, $K = 0.5$. **(07 Marks)**

c. Solve numerically the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0, t) = 0 = u(1, t)$, $t \geq 0$ and $u(x, 0) = \sin\pi x$, $0 \leq x \leq 1$. Carryout computations for two levels taking $h = \frac{1}{3}$ and $K = \frac{1}{36}$. **(06 Marks)**

8 a. Find the Z-transforms of :
i) $\cos n\theta$ ii) $\sin n\theta$. **(06 Marks)**

b. Find the inverse z – transform of :
$$\frac{4z^2 - 2z}{z^2 - 5z^2 + 8z - 4}.$$ **(07 Marks)**

c. Solve the difference equation :
$u_{n+2} + 3u_{n+1} + 2u_n = 3^n$ with $u_0 = 0$, $u_1 = 1$
using Z-transforms. **(07 Marks)**

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